

Homework #9
ECEN/MAE 5513

Consider a communication system, in which a signal X is transmitted ten times:

$$Y_i = X + E_i \quad i = 1, 2, \dots, 10$$

where the E_i are independent Gaussian random variables, with mean zero and variance 1, which represent the noise in the communication channel, and the Y_i are the received signals. Your objective will be to estimate the transmitted signal X from the received signals Y_i .

1. Assuming that X is an unknown constant, find the joint density function of the received signals. Find the likelihood function and the maximum likelihood estimate of X -- \hat{X}_{ML} . Make a plot of the likelihood function. Find the variance of \hat{X}_{ML} .
2. Now assume that X is a Gaussian random variable, with mean 2 and variance 0.1. Find the minimum mean square error estimate of X -- $\hat{X}_{MS} = E[X|Y_1, Y_2, \dots, Y_{10}]$. As part of this step, you will need to find the posterior density function and the mean of the posterior density function. You should also identify the variance of the posterior density function, which you will use later. Make plots of the prior and posterior densities.

Test your answers from Parts 1. and 2. using Monte Carlo simulation. Use 100,000 Monte Carlo trials. For *each* trial, generate *one* X , using the Matlab function `randn` (modified appropriately to have the correct mean and variance). Then, generate ten E_i , using `randn`, and calculate ten Y_i . Then, calculate \hat{X}_{ML} , \hat{X}_{MS} , and errors $\tilde{X}_{ML} = X - \hat{X}_{ML}$ and $\tilde{X}_{MS} = X - \hat{X}_{MS}$. You should have a total of 100,000 values of these four variables and 100,000 values of X saved at the completion of your Monte Carlo trials. You will need them for the remaining parts. (Before you start your first Monte Carlo trial, issue the Matlab command `rng(myID)`, where `myID` is your OSU ID #.)

3. After the 100,000 trials, find the average squared error for both \hat{X}_{ML} and \hat{X}_{MS} . Which estimate is better? Explain this result. If you had only transmitted X two times, how do you think this result would change? What if you transmitted X 100 times? Explain.
4. Make histograms (use at least 40 bins) of X , \tilde{X}_{ML} and \tilde{X}_{MS} . Compare the histograms of X and \tilde{X}_{MS} with the plots of the prior and posterior density functions, respectively, that you found in part 2. Discuss and explain the similarities and differences.
5. Use the Matlab command `std` to find the estimated standard deviations of \tilde{X}_{ML} and \tilde{X}_{MS} . Compare the estimated standard deviation (std) for \tilde{X}_{MS} with the std for the posterior density function you found in Part 2, and the estimated std for \tilde{X}_{ML} with the theoretical value from Part 1. Discuss and explain the similarities and differences.
6. Run additional Monte Carlo simulations with the following changes. Try variances for X of 0.1, 0.5 and 10. Try transmitting X 2, 10 and 100 times. For all combinations, explain how the accuracies of \hat{X}_{ML} and \hat{X}_{MS} change. Provide a discussion of the effects of the variance of X and the number of transmissions of X on the performance of these two estimates, both absolutely and relative to each other.