Consider the following function of two variables:

\[ F(x) = e^{(2x_1^2 + 2x_2^2 + x_1 - 5x_2 + 10)} \]

i. Find the second-order Taylor series approximation for \( F(x) \) about the point \( x = [0 \ 0]^T \).

ii. Find the stationary point for this approximation.

iii. Find the stationary point for \( F(x) \). (Note that the exponent of \( F(x) \) is simply a quadratic function.)

iv. Explain the difference between the two stationary points. (Use MATLAB to plot the two functions.)

For the following functions find the first and second directional derivatives from the point \( x = [1 \ 1]^T \) in the direction \( p = [-1 \ 1]^T \).

i. \( F(x) = \frac{7}{2}x_1^2 - 6x_1x_2 - x_2^2 \)

ii. \( F(x) = 5x_1^2 - 6x_1x_2 + 5x_2^2 + 4x_1 + 4x_2 \)

For the functions of Exercise 2:

i. find the stationary points,

ii. test the stationary points to find minima, maxima or saddle points,

iii. provide rough sketches of the contour plots, using the eigenvalues and eigenvectors of the Hessian matrices, and

iv. plot the functions using MATLAB to verify your answers.

For the functions given in Exercise 2 perform two iterations of the steepest descent algorithm with linear minimization, starting from the initial guess \( x_0 = [1 \ 1]^T \). Write MATLAB M-files to check your answer.

Repeat Exercise 4 using the conjugate gradient algorithm.

Problem 4.5 from Spooner.

Problem 4.9 from Spooner.